Use of a stochastically enhanced framework for the modeling of uncertain dynamical systems

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ABSTRACT: The goal of the present study is to propose a structural identification framework able to exploit both vibrational response and environmental information data in extracting structural models, which describe structural behavior for its complete operational spectrum. In doing so, it is imperative to devise a framework able to distinguish between variations of structural properties, which may be attributed to operational effects, and therefore lie within regular structural condition bounds, versus variations which indicate short- or long-term damage effects. The computational tool developed herein for proper decomposition of such effects is based on polynomial chaos expansion. This paper further illustrates how the introduced method may be incorporated into a damage detection framework. The method is assessed via implementation on a set of real world applications, revealing a robust condition assessment tool.

1 INTRODUCTION

In recent years, Structural Health Monitoring (SHM) of engineering structures has developed into an important area of research, further justified by the timely realization of an ageing and deteriorating infrastructure demographic. SHM constitutes a broad term, which by large also deals with the online detection and identification of structural damage and therefore holds a critical significance for the case of large-scale civil structures, where a failure incidence is tied not only to significant financial losses but also to potential human toll, in a worst-case scenario.

Particularly, to what non-destructive damage detection is concerned, vibration-based methods have evolved into one of the fastest growing research areas within the SHM field [1]. However, such methods are still far from being successfully implemented in large-scale civil structures due to a multiplicity of reasons, including lack of precise load measurements, since the usual source of excitation is ambient loading, as well as due to the susceptibility of these structures to uncontrollable influences, such as solar irradiation, temporal temperature gradients, humidity, and others [2], [3]. Even though the former problem is adequately treated today by a number of accurate and robust output-only (operational) identification methods [4], the latter is still under research and in focus of a number of recent studies.

Within this context, Alampalli [6] identifies natural frequency shifts of up to 50% for an abandoned bridge in Claverack (NY) and attributes this variation to the freezing of the bridge supports. Farrar et al. [5] report a variation of the eigen-frequencies of the Alamosa Canyon Bridge of approximately 5% over a 24-hour period due to varying spatial temperature gradients. Maximum differences varying from 14 to 18% were also calculated for the first four natural frequencies of the Z24 bridge, monitored for the SIMCES project [4]. Catbas et al. [7] found that the structural reliability of a long-span truss bridge in USA is highly affected by the ambient temperature, while Cross et al. [8] investigated the effect of temperature, traffic loading and wind speed on the modal properties of the Tamar (UK) suspension bridge and identified differences up to almost 5%. Finally, Yuen and Kuok focused on the description of the relationship between natural frequencies and environmental factors (temperature and humidity) for tall buildings [9].

The approaches implemented within such a framework may be primarily classified in two categories: i) methods which try to remove the environmental signatures from vibration measurements or estimated damage indices (e.g. modal properties) [10], and ii) methods which try infer a functional dependence between the measured vibration data and/or the extracted structural properties with respect to measured environmental quantities [4], [9]. This study follows the second approach, which

is capable of providing additional insight into the mechanism of the variation of structural properties under operational conditions, even for potentially unobserved sources of influence.

The proposed method is structured as follows. In a first step, when dealing with stationary systems (commonly bridges and buildings will be regarded as such), conventional operational modal analysis methods may be employed in order to identify the modal characteristics of the healthy structure. It should be noted that in dealing with systems that are non-stationary in nature, such as Wind Turbine facilities, time varying methods may correspondingly be implemented, for tracking the characteristics of these systems [16]. In a second (training) phase a polynomial chaos expansion (PCE) [11] tools is employed for describing the dependences on measured environmental conditions via a functional representation. Finally, in a final stage, following the training phase, the PC-based estimated statistical properties of the modal characteristics may be used for condition assessment and potential damage detection.

In order to illustrate the workings of the method, the proposed framework is applied on a series of data from actual structural systems, including the ÜF Bärenbohlstrasse Bridge in Zürich, as well as the benchmark SHM problem of the Z24-bridge in Switzerland [4], [12]. The results of these studies demonstrate a tool, which may reliably be employed into practice for condition monitoring of large-scale real world structures operating in a wide range of conditions.



Figure 1. Schematic diagram of the proposed identification method.

2 STRUCTURAL IDENTIFICATION FRAMEWORK

As already mentioned, the introduced SHM method aims at a global representation of a structural system, in the sense that it may accurately describe the dynamic properties of the structure for a range of operational conditions. The devised framework essentially comprises a long-term monitoring approach, since it requires a set of training data incorporating a cycle of the variation of the influential (input) quantity. The approach additionally exploits so-called short-term monitoring methodologies for the instantaneous extraction of structural properties, i.e., of the influenced (output) quantities. The latter refers to the implementation of conventional operational modal analysis methods for extracting the modal characteristics of the healthy structure. Once these are attained, a polynomial chaos expansion (PCE) method [11] is further applied for the projection of these estimates on the probability space of the measured environmental conditions (training phase).

The PCE tool operates on the assumption of independent input variables. However, when discussing environmental conditions (temperature, humidity, wind direction and velocity, and others), the common case is that a large bulk of data is in fact available, which is in some degree correlated. In order to select only a small number of independent features, which is nonetheless able to describe a large portion of this data, an Independent Component Analysis (ICA) algorithm [13] is utilized herein, as briefly explained in a later section. A schematic diagram of the training phase of the proposed framework is illustrated in Figure 1. Following the training phase, the PCE-based estimated statistical properties of the modal characteristics may be utilized for tracking the system's condition and devising damage indicators.

2.1 Polynomial Chaos Expansion (PCE)

A short overview of the employed mathematical framework is provided in this section. For further details the interested reader is pointed to previous works of the authoring team [17][18][19]. PCE concerns the expansion of a random output variable on polynomial chaos basis functions, which are orthonormal to the probability space of the system's random inputs. Consider a system S comprising M random input parameters adequately represented by a set of independent random variables $\{\Xi_1, ..., \Xi_M\}$. The latter set may pertain to e.g. temperatures measured at different locations of the structure, gathered in a random vector Ξ of prescribed joint Probability Density Function (PDF) $p_{\Xi}(\xi)$ [11]. The resulting structural output denoted by $Y = S(\Xi)$, and described for instance via natural frequency estimates, will also be random. Provided that Y has finite variance, it can be expressed as follows:

$$Y = \mathcal{S}(\mathcal{Z}) = \sum_{d \in \mathbb{N}^M} \theta_d \varphi_d(\mathcal{Z}) \tag{1}$$

where θ_d are unknown deterministic coefficients of projection, d is the vector of multi-indices of the multivariate polynomial basis, and $\varphi_d(\Xi)$ are the polynomial basis (PC) functions orthonormal to $p_{\Xi}(\xi)$. These basis functions $\varphi_d(\Xi)$ result as tensor products of the corresponding univariate functions [11]. Each probability density function may be associated with a well-known family of orthogonal polynomials. For instance, normal distribution is associated with Hermite polynomials while uniform distribution with Legendre (Table 1). A list of the most common probability density functions along with the corresponding orthogonal polynomials and the relations for their construction may be found in [14].

Table 1.Types of Wiener-Askey polynomial chaos and their underlying random variables.

PDF	Support	Polynomials
Normal (Gaussian)	$(-\infty,\infty)$	Hermite
Gamma	$[0,\infty)$	Laguerre
Beta	[0,1]	Jacobi
Uniform	[-1, 1]	Legendre

For implementation purposes, these basis functions series need be truncated to a finite number of terms, with the usual approach being the selection of the multivariate polynomial basis with total maximum degree $|\mathbf{d}_j| = \sum_{m=1}^{M} d_{j,m} \leq P$ for every *j*. When truncating the infinite series of expansion of Equation (1) to the first *p* terms, the resulting PCE model is fully parametrized in terms of a finite number of deterministic coefficients of projection θ_d . The parameter vector θ_d may be estimated by solving Equation (1) in a least squares sense. Toward this end, the data of the output variables and the PDFs of the input variables have to be employed. The PDFs of the input variables may be obtained by fitting known statistical distributions to the observed input variables values.

Considering for example a structural system subjected to ambient excitation which is susceptible to changes of the environmental conditions, a given output variable, such the estimates of its first natural frequency, could be expanded on a PC basis constructed to be orthogonal to the experimentally estimated PDF of temperature data. In this way, an indication is obtained on the sensitivity of the natural frequencies of the structure to the probabilistic properties of temperature variations affecting the structural system.

2.2 Independent Component Analysis (ICA)

ICA is a source separation method, which aims at estimating independent unobservable (latent) variables that are intermixed with observed quantities [13]. This is herein required for sorting out independent input variables among the possibly numerous measurements of temperature and humidity that are potentially available via a dense grid of corresponding environmental sensors. The key of the method lies in its search for non-Gaussian components, in contrast to principal component analysis and other second order methods, which are based on the covariance matrix of random variables. To

this end, ICA combines a static linear mixture model with higher order statistics in order to identify unobservable variables that are as non-Gaussian as possible. The measure of non-Gaussianity may be based on kurtosis, negentropy, and others [13]. The details of the method may be found in [19], while a flowchart of a typical ICA algorithm implementation is shown in Figure 2.



Figure 2. Flowchart of the ICA algorithm.

Finally, it should be added that the characteristics of the ICA-based are particularly attractive within the context of the proposed SHM framework. Significant discussion is nowadays allocated regarding the storage potential of the Big Data stream resulting from long-term SHM implementations. The advantage delivered by the ICA lies in the reduction of latent variables to be regarded as input to the PCE algorithm, i.e., a reduction of the data to be stored form the SHM system by extracting only a small number of valuable input and output features.

3 APPLICATION CASE STUDIES

The workings of the method will be now demonstrated via implementation on a set of actual largescale structures under both operating and damaged conditions.

3.1 Implementation on Systems under Operational Conditions

As part of a project funded by FEDRO on "Structural Identification for Condition Assessment of Swiss Bridges", the Überführung Bärenbohlstr. bridge has been chosen as a suitable test case due to its dimensions, static system, and ease of access (Figure 3). According to existing documents, the bridge was designed in 1977 and construction was completed in 1980. This implies that the bridge is currently at 34 years of age, therefore not extremely aged. The bridge was monitored from July 2013 till July 2014 through a permanent system acquiring hourly ambient vibration response and environmental condition measurements. More specifically, 18 acceleration sensors measuring ambient vibration responses (vertical axis), and two environmental condition sensors for measuring air temperature and humidity were installed. After the twelve-month period, more than 6500 datasets were collected. For these datasets, the estimation of the modal properties of the structure based on the NExT-ERA method was realized. The method hyper-parameters (state space model order equal to 30; 80 Markov parameters for the construction of the Hankel matrix) were kept constant for all datasets and they were selected based on initial modeling of a randomly selected training set of vibration response data.



Figure 3. Test-case: Bärenbohlstr. bridge.

The first four natural frequencies estimated through NExT-ERA method are selected as the output variables for the SHM methodology. These estimates are plotted versus time axis Figure 4. It may be observed that all four natural frequencies vary with time in three different patterns: i) almost linear increase from July 2013 till end of November 2013, ii) high variability from end of November 2013 till end of December 2013, and iii) linear decrease from January 2014 till July 2014 when they seem to actually return to their July 2013 values.



Figure 4. The first four identified natural frequencies of the bridge for the various datasets of the long-term monitoring system.

Certainly, this variability may be attributed to the seasonal variation of the environmental conditions. This dependency is clearly depicted in Figure 5 where the natural frequency estimates are plotted versus temperature and humidity. A bilinear temperature – natural frequency relationship may be observed between negative and positive temperatures while a more complex relationship seems to characterize the dependency of the identified natural frequencies and humidity.

Even though, the list of input variables affecting the structural behavior of the bridge is not necessarily limited to the measured environmental sources, it is assumed that these are herein adequate for describing a large percentage of the variability of the estimated modal properties. Spatially distributed temperature sensors could be employed in order to additionally capture spatial gradients of temperature in order to increase the information related to the operational conditions of the bridge.



Figure 5. The first four identified natural frequencies versus (a) temperature; (b) humidity.

In order to expand the natural frequency estimates onto a polynomial chaos basis orthogonal to the PDFs of the random input variables, i.e., temperature and humidity, their independence has to be checked. Looking at the scatter plot of Figure 6a, it is evident that these are highly correlated (correlation close to 0.75). Therefore, the ICA has to be implemented before PCE is applied on the natural frequency estimates. The scatter plots of the corresponding ICA-based estimates of the independent random (latent) variables are shown in Figure 6b, with the variables indicating a correlation close to 0. It should be noted that these transformed variables do not correspond to temperature and humidity anymore.



Figure 6. Histograms of (a) the measured temperature and humidity; (b) the latent variables occurring after application of the ICA, along with corresponding scatter plots.

The identified modal frequencies are then expanded onto the probability space of the latent input variables through PCE. Toward this end, the input variables are transformed into uniformly distributed variables by using their non-parametrically estimated cumulative distribution functions and PC basis consisting of multivariate Legendre polynomials is finally calculated.

The set of natural frequency estimates is divided into estimation and validation set. The estimation set comprises of the first 1250 values corresponding to the first five months of the monitoring period till early December 2013, while the rest of the values are used as validation set, that is 1114 values. The estimation set values are expanded on a PC basis consisting of multivariate Legendre polynomials of maximum order five, that is a total number of 21 basis functions, while it was observed that increasing the maximum order further did not significantly increased the accuracy of expansion.

It is be observed that the PCE-model estimates are capable of reliably reproducing the estimation set values, while more importantly very good accuracy is achieved for values lying in the validation set, including the more challenging winter months of the monitored interval (Figure 7).



Figure 7. PCE natural frequency estimates for both the estimation and validation set contrasted to the true NExT-ERA based estimates.

Nonetheless, damage detection still comprises a difficult decision, which has to be based on multivariate statistics relating to the PCE expansion errors for the four natural frequency estimates. In order to simplify this problem, the ICA algorithm is utilized for estimating a reduced number of independent random variables from the estimated natural frequencies (output variables). As a result, a single *damage (or condition) index* is extracted relying on a univariate statistical hypothesis testing for the characterization of the "health" state of the structure. By implementing the ICA once again, this time retaining a single latent output variable, the results shown in Figure 8 are attained, corresponding to a simple one-dimensional damage indicator. Based on the statistical characterization of the PCE model prediction errors, confidence intervals may be calculated with respect to this simplified metric, and used for damage detection as described before.



Figure 8. Extracted feature variable compared to the PCE modeling estimates (top) and the extracted damage index based on the PCE prediction errors for the Überführung Bärenbohlstr.

The strength of the methodology presented herein therefore lies in the fact that it is able to account for the influence of exogenous factors, such as environmental or other operational conditions, upon structural behavior without use of a detailed numerical structural model. Instead, this is a tool easily generalizable for any type of structure, without necessitating a-priori knowledge (or drawings) of the structure itself, but merely a training interval during which the structure is monitored. Hence, such a process is only implementable within a long-term monitoring perspective.

3.2 Implementation on Systems under Damaged Conditions

In already published work [19], the proposed SHM framework has already been validated using test data from another Swiss bridge, i.e., the Z24 benchmark case [12]. The Z24 bridge was monitored for a period of 9 months and finally "artificially damaged" for the purposes of the BRITE-EURAM SIMCES project. This bridge was over-passing the A1 Zurich-Bern highway connecting Utzemstorf and Koppigen and its demolition was planned after the monitoring period because a new railway, adjacent to the highway, required a new bridge with a larger side span. The bridge was monitored from November 1977 till August 1998 through a permanent system acquiring hourly ambient vibration response and environmental condition measurements. More specifically, 16 acceleration sensors measuring ambient vibration responses (13 of them are shown in Figure 9a, while three more were installed on one of the piers), while 49 environmental condition sensors were installed for measuring air temperature, wind characteristics, humidity, bridge expansion, soil temperatures at the boundaries and bridge concrete temperatures. Data from these sensors were acquired hourly.



Figure 9. (a) The Z24-Bridge: longitudinal section and top view [15]; (b) Types of artificial damages induced on bridge Z24 [11].

After the nine-month period, progressive damages were induced artificially within a month period (August 10 - September 11, 1998). The monitoring system was still in operation while during night vibration tests based on hammer, dropping mass device and shakers were performed. The specific types of induced damages are summarized in Figure 9b, while more details on the damage scenarios and description of the simulation of the real damage cause may be found in [12]. The framework described in this work has been applied on the monitored data from the Z24 bridge. Specifically, the temperatures measured at six locations at the center of the middle span along with the air temperature are used, while the first four natural frequencies estimated through Stochastic Subspace Identification (SSI) and an automated operational analysis method are considered as the output of the structural system.

The dependency of natural frequency estimates on the temperature measured at the center of the web is shown in Figure 10 indicating a bilinear temperature – natural frequency dependence. Interestingly, the variation that is due to environmental conditions is in fact more pronounced than the one attributed to the damages. A robust predictor tool therefore necessitates incorporation of these effects in the simulation process.

Using the PCE method described above, which is capable of incorporating the effect of temperature, a condition indicator is extracted. Using the PCE method described above, which is capable of incorporating the effect of temperature, a condition indicator is extracted. Figure 11 summarizes the results of PCE approach. The upper plot of the figure illustrates:

(a) A quantity extracted from the actual measurements (blue marker). This "feature" is linked to the monitored natural frequencies. It is extracted from these 4 quantities, via use of the ICA algorithm, as a condensed single salient feature.

(b) The training set (red marker), comprises 1500 values randomly selected from the first eight months of the monitoring period. It is essential that the training set includes a full seasonal cycle and therefore this framework is only meaningful within a long-term monitoring context.

(c) The prediction of the PCE model (green marker). As long as the model prediction agrees with the measured quantity (in blue), the structural response lies within operational (regular) bounds. Deviations of the predicted from the measured quantity serve as indications of irregularity or damage.



Figure 10. The first four natural frequency estimates plotted against temperature (center of the web).

A better way to quantify this however is via the plot of the error illustrated in the lower graph of Figure 11.



Figure 11. Extracted feature variable compared with the PCE modeling estimates (top) and the extracted damage index based on the PCE prediction errors for the Z24 benchmark case, where artificial damage is inflected after 9 months of monitoring (vertical line).

During regular operation, this error should be uniformly distributed within a threshold defined by the horizontal dashed lines. Indeed prior to the occurrence of damage (marked with a black vertical line), the error lies within the prescribed thresholds, apart from isolated outliers, which can be ignored as isolated points. In the period following the point where the 1st damage is induced the error plot develops a persistent offset from the expected mean, exceeding the prescribed thresholds, which correctly translates to damage indication. In fact, the method is able to detect, also the preliminary works performed on the structure in preparation of the damage sequence, consisting in the lowering of the piers on August 7th, i.e., 3 days before the actual implementation of the designed damage scenarios (August 10th), which is marked in the Figure with a red vertical line.

The PCE-model predictor tool and the extracted performance index is therefore capable of reliably reproducing the estimation set values, while more importantly a very good accuracy is also achieved for values belonging in the validation set for the interval prior to damage.

3.3 Discussion and Open Ended Questions

The task explored so far was that of mere damage detection. The developed methodology can be further exploited to quantify damage, since damages of different intensity can be associated with a different trend in the offset of the condition index from pre-scribed thresholds. The Z24 case, does not allow us to proceed with a further investigation, since the data made available, through our collaboration with Prof. E. Reynders and Prof. G. De Roeck from KU Leuven, comprise processed information in the form of natural frequency estimates over time. Instead, further data with respect to modal shapes would be essential when discussing damage quantification, since the impact of damage on modal shapes is more pronounced than its impact on the frequency content.

Naturally, it needs to be stressed that the type of damage this PCE based method promises to identify are only those which bear an impact on the vibrational response of the structure. As is the case for non-destructive evaluation methods, this technique has a specific range of implementation and should be correspondingly used. The end goal should be the efficient coupling of available approaches for achieving the maximal benefit for infrastructure maintenance and management.

4 CONCLUSIONS

Environmental condition data, nowadays largely available in modern SHM systems of large-scale civil engineering structures, should be accounted for in favor of identifying comprehensive dynamic models. To this end, a long-term monitoring framework is introduced, relying on a polynomial chaos expansion tool for the accurate description of the effect of stochasticity stemming from environmental factors onto structural response. A condition index is delivered that can serve as warning of irregularity/damage on the bridge and which takes environmental influence into account. This component requires the operation of a low-cost monitoring system over a long period of time but can provide significant information concerning the influence of environmental factors on structural performance, by simply relying on ambient (operational) response data. Once this influence is quantified, a simple indicator may be extracted able to indicate whether the structure's condition lies within regular bounds or whether, to the contrary, a deviation indicating damage, or deterioration is tractable. Application of the method on the two actual large-scale bridge case studies demonstrates the potential of adoption of the proposed framework into actual practice.

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