

VIBRA-1-2-3: Validation of a model for railway vibration prediction

After 15 years of vibration measurement in buildings near railway lines a considerable amount of data has been accumulated. During this time period the program VIBRA-1-2-3, i.e. a program for the prediction of railway vibrations in buildings - has been developed jointly with Swiss Rail. It's time now to compare the prediction algorithm in the VIBRA-model with the measurement data in order to see the validity and the shortcomings of the model used.

As an introduction the railway vibration problem as such is described briefly. Also some basic principles from physics are recalled. They will help us to understand better the mechanisms involved in the context of railway vibrations.

Then two models will be discussed: The simple model in VIBRA-1, that neglects the frequency dependence and the spectral model of VIBRA-2, where the frequency dependence is taken into account by means of third octave source and transfer spectra. For the first (simple) model a comparison will be presented between predicted and measured data. Based on these comparisons means to improve the model will be proposed. For the second (spectral) model representative source and transfer spectra gained from statistical evaluation of a large amount of data will be presented. A validation of the second (spectral) model will and cannot be discussed. This model should be used for these cases where part of the data required, like the source spectrum or some of the transfer spectra, can be measured on site. If this is not possible, i.e. if no data on the source and on the receiver are available, we can as well restrict our effort to using the simple model in VIBRA-1.

1 The Railway Vibration Problem

Railway vibrations have such a complex character and appear in most different configurations that it is almost impossible to come up with a prediction model that can be used generally. Still, if we accept the inherent uncertainties and do not overstress the prediction capabilities these models can be quite useful, especially if the geometric configuration do not deviate too much from the standard case shown below.

The “standard case” consists of a railway line and, at a certain lateral distance, a building approximately on the same level as the railway line. The problem at hand can be broken into five separate tasks:

- the definition of the vibration source,
- the attenuation of the vibration in the free field between the track and the building,
- the coupling between the soil and the building,
- the floor vibrations and
- the radiation of secondary noise in the room.

If we are able to come up with an adequate model for each of these five parts the problem should actually be solved.

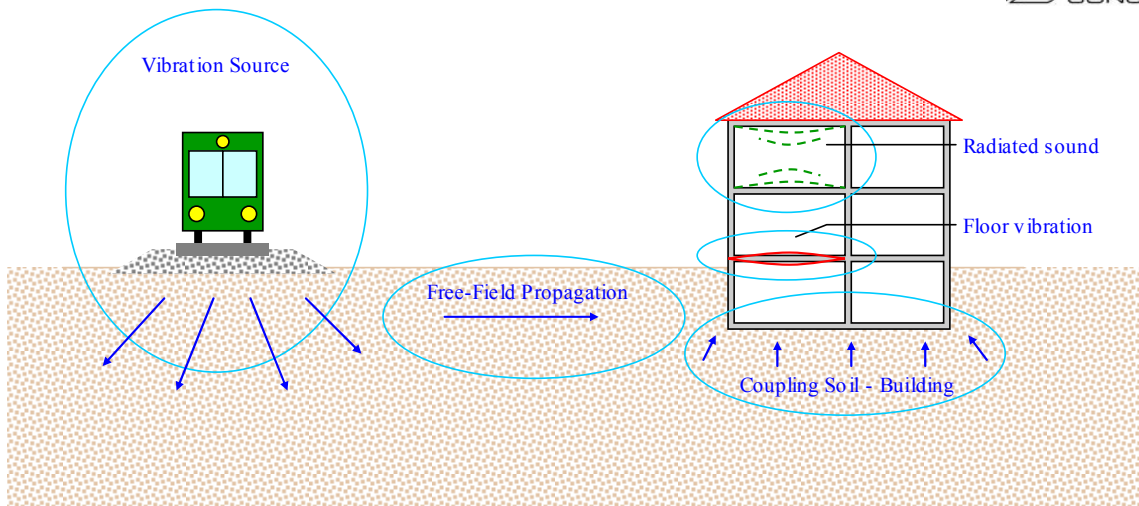


Fig. 1.1 Configuration of the “Standard case”

In reality quite often “special situations” complicate our work and the models we have developed for the standard case are not more than a poor approximation.

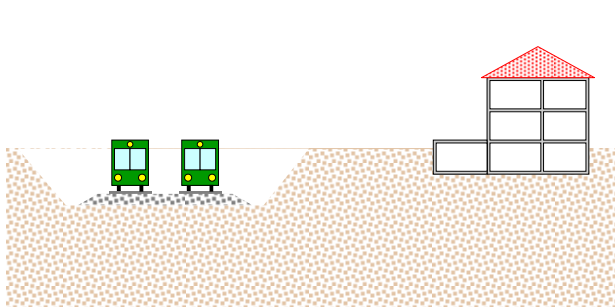


Fig. 1.2 Special case “Railway line in a cut”

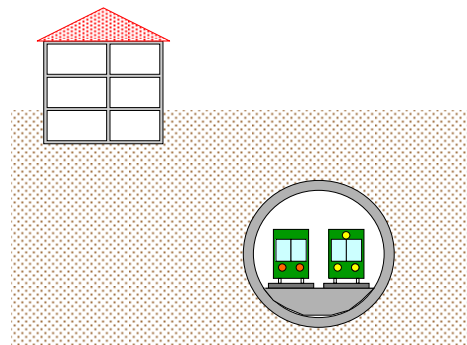


Fig. 1.3 Special case “Tunnel”

2 Some basic principles from physics

The five tasks described above have been treated – at least in their ideal configuration – by scientists more than hundred years ago and it might be quite helpful to look at these solutions in order to better understand the railway vibration problem.

2.1 Wave propagation in an elastic half-space

In an elastic half-space we distinguish between body waves and surface waves. The first type of waves expands laterally and into the depth of the half-space while the second type expands only laterally. We also distinguish between point sources and line sources. Especially line sources might attract our attention as a train might best be modelled as a line source.

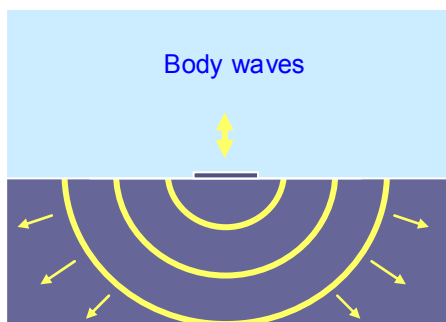


Fig. 2.1a Propagation of body waves

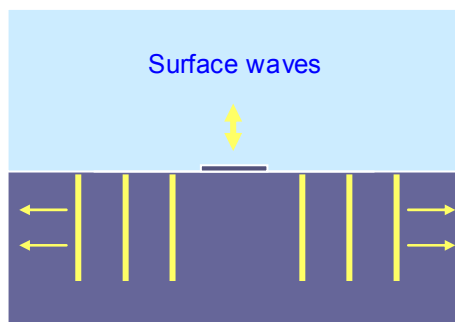


Fig. 2.1b Propagation of surface waves

Purely from the point of view of energy conservation we arrive at the attenuation laws given in table 2.1 or shown in Fig. 2.2.

Tab 2.1: Propagation in an elastic half-space

Body wave		Surface wave	
Point source	Line source	Point source	Line source
$v = v_0 \left(\frac{r_0}{r} \right)^2$	$v = v_0 \left(\frac{r_0}{r} \right)^1$	$v = v_0 \left(\frac{r_0}{r} \right)^{0.5}$	$v = v_0 \left(\frac{r_0}{r} \right)^0$

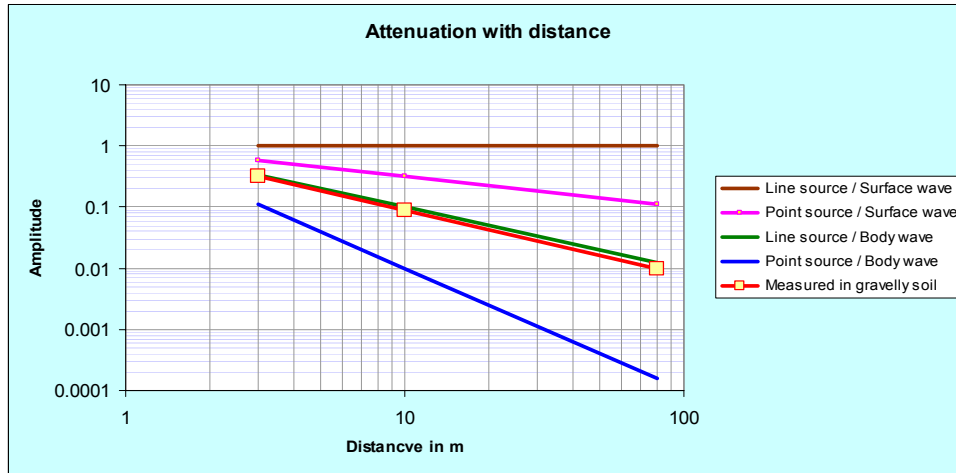


Fig. 2.2 Attenuation of body waves and surface waves for point and line source

Comparing the line for measured data with the lines for the four equations given in table 2.1 (see Fig. 2.2) we might expect that the body waves from a line source might yield the best fit. However this is misleading: The material damping has not yet been taken into account. As with a SDOF-system that exhibits an exponential decay of amplitude with time, the soil shows an exponential decay with time and distance from the source with higher damping for higher frequencies as shown in Fig. 2.3.

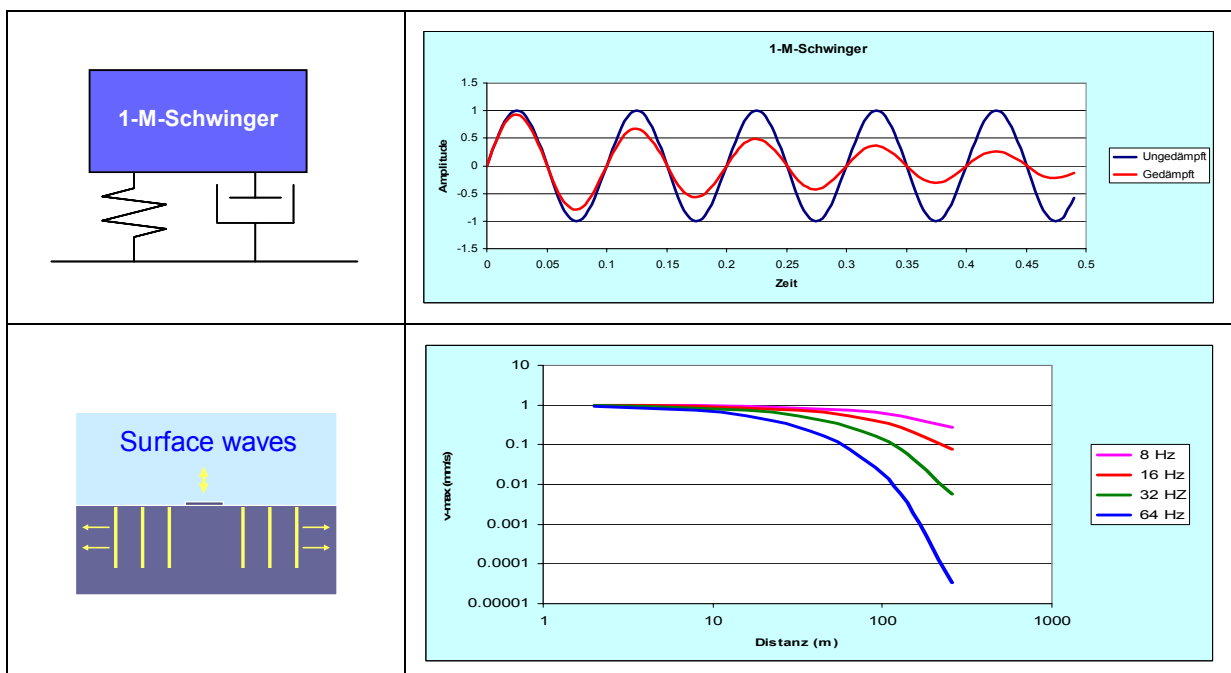


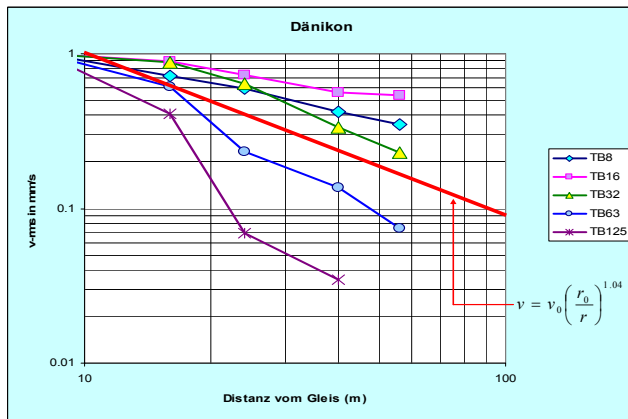
Fig. 2.3 Atténuation des vibrations et des ondes de surface

The corresponding equation, which can be easily derived from the SDOF-system, is given as:

$$v = v_0 \left(\frac{r_0}{r} \right)^{n(f)} e^{\left(-\frac{2\pi f D}{v_B} (r-r_0) \right)} \tag{2.2}$$

It introduces a frequency dependent exponent $n(f)$ and a second term containing the material damping D of the soil and the surface wave velocity v_s . However, if we consider for instance the data in Fig. 2.4, we might be tempted to use in place of Eq. (2.2) the more amenable Eq. (2.3), where we have only the term $n(f)$ to derive statistically from our data.

$$v = v_0 \left(\frac{r_0}{r} \right)^{n(f)} \tag{2.3}$$



Freq. Hz	n(f)
8	0.54
16	0.31
32	0.76
64	1.34
128	2.17

Fig. 2.4 Spectral attenuation measured in a gravelly soil

2.2 Coupling between soil and building

A house near a railway line might – in its most simple form – be modelled as a rigid block on a half-space. The rigid block on its turn can be modelled as a mass-spring-dashpot system with frequency dependent stiffness and damping. Thus using the well known theory for machine foundations we can obtain some insight into the coupling between soil and buildings. Taking for instance two types of buildings, a small house and a multi-storey building as shown in Fig. 2.5, we obtain the parameters given in table 2.2. The values for L (length) and W (width) and the mass M are assumed values. The values for frequency f and damping D are obtained from the theory of the rigid block on an elastic half-space. From the frequency and damping values we can then calculate the amplification (or transmissibility) of a mass-spring-dashpot system resting on a vibrating base as shown in Fig. 2.5 (right).

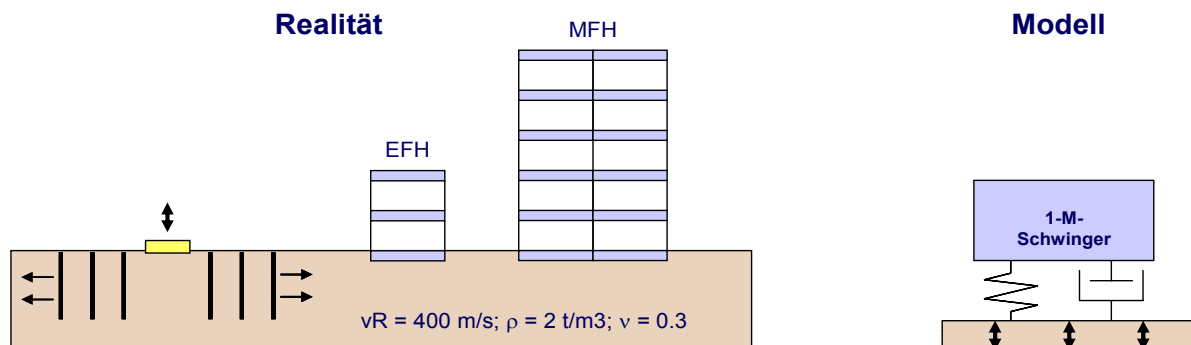


Fig. 2.5 Modelling the coupling effect with a mass-spring-dashpot system

Tabelle 2.2 Parameters for the SDOF-system

	House	MS Building
L x W	10 x 10 m	20 x 20 m
M	250 t	8000 t
f	20 Hz	5 Hz
D	1.02	1.44

The interesting part of this model is that it compares surprisingly well with the measured data shown in Fig. 2.6. It explains also why we may - under certain conditions - have an amplification of the free-field vibration through the coupling effect and why buildings at longer distance from the track exhibit higher coupling factors.

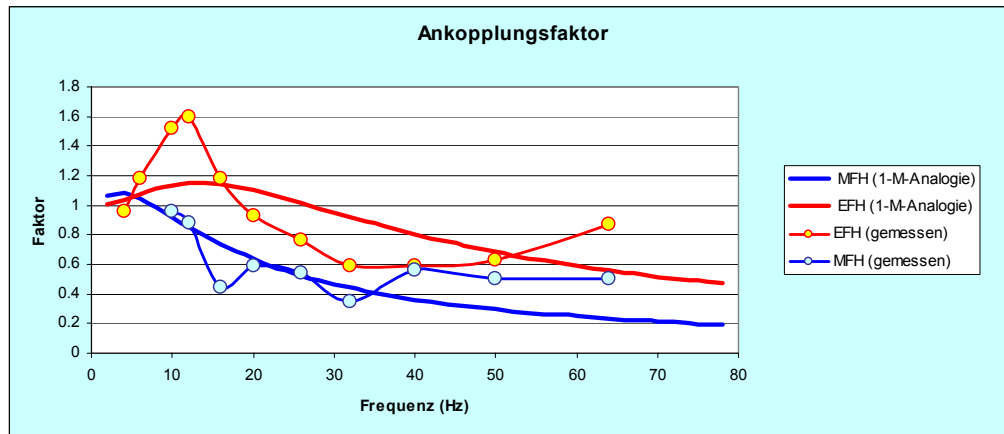


Fig. 2.6 Comparison of the amplification function for a SDOF-system with measured data

2.3 Attenuation over building height

Attenuation of vertical vibrations over building height should – at least in a regular structure – follow the same law as we know for instance from the propagation of waves in a damped shear beam. Due to the damping the amplitudes decay in an exponential fashion as given in Eq. (2.4):

$$V_n = V_0 \cdot e^{-n\xi} \tag{2.4}$$

with V_n = vibration amplitude on floor level n and V_0 = vibration amplitude on ground floor. Measurements in a 12 storey building (see Fig. 2.7) show that an exponent $n = 0.1$ yields quite a good approximation.

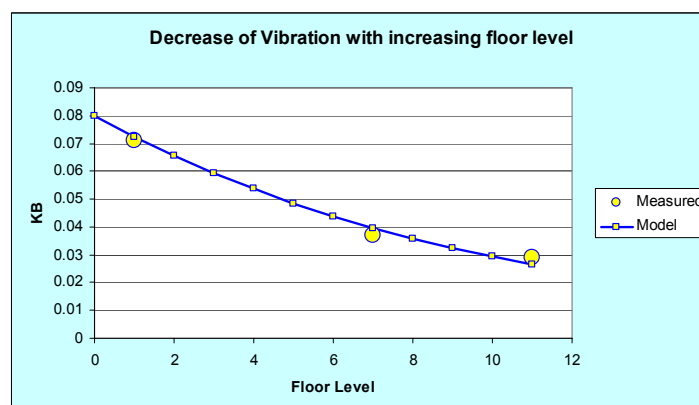


Fig. 2.7 Decrease of vibration with increasing floor level

2.4 Floor vibrations

The eigenfrequencies of a regular square floor with simply supported boundaries are found in most textbooks on structural dynamics and are given by Eq. (2.5).

$$f_{ij} = \frac{\lambda_{ij}^2}{2\pi a^2} \left[\frac{Eh^3}{12\gamma(1-\nu^2)} \right]^{1/2} \quad (2.5)$$

Evaluating this equation for three typical concrete floor sizes and assuming a damping of 5 % we find – as shown in Fig. 2.8 - that the dominant frequencies due to the first eigenmode are in the range of 25 to 50 Hz and between 50 to 100 Hz due to the second eigenmode. The amplification factor should be expected to be in the range of 10. As can be seen in Fig. 2.9 the amplification of 10 is quite realistic. It should be noted however that this amplification holds only for the limited range of resonance frequency. The overall amplification factor is in the order of 3.

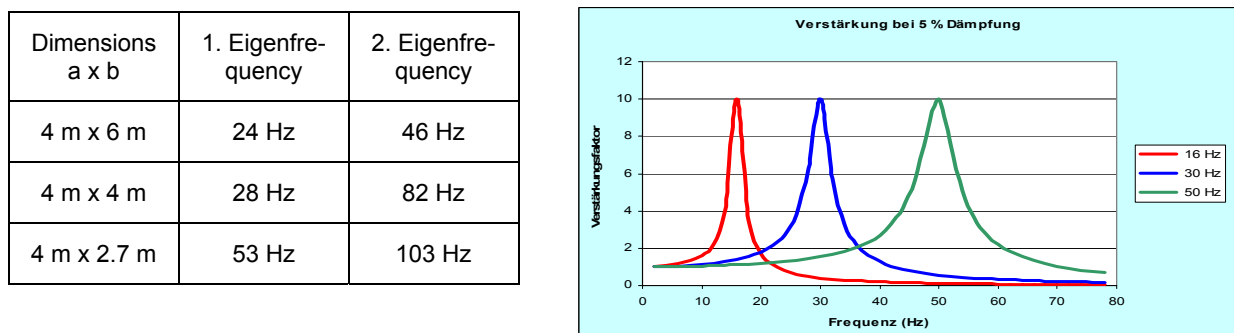


Fig. 2.8: Eigenfrequencies and resonance curves for three different plates

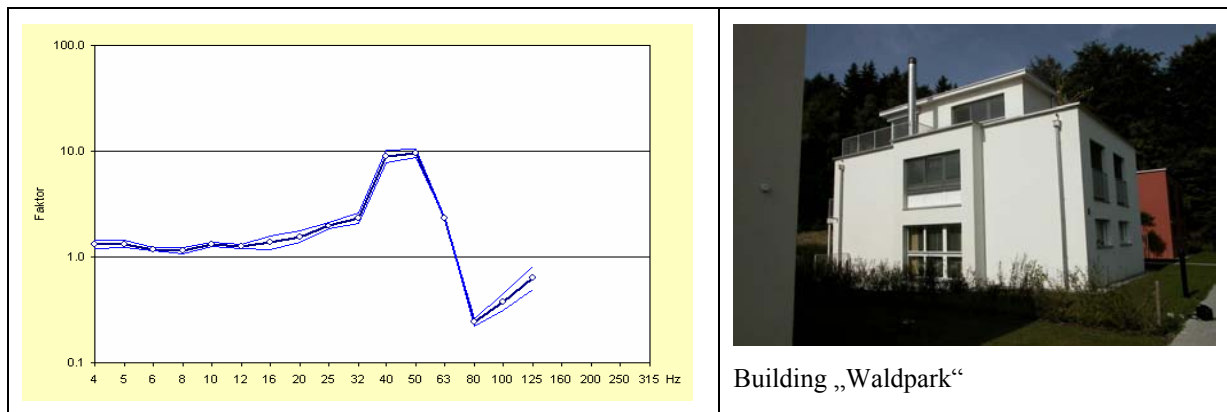


Fig 2.9: Transfer spectrum between foundation and floor in the building „Waldpark“

2.5 Radiated sound

The secondary radiated sound (or structure borne noise) is created by the vibration of the floor, the ceiling and the walls. The vertical movement of the floor sends a pressure wave into the air which – if it is in the right frequency range – will be perceived as noise.

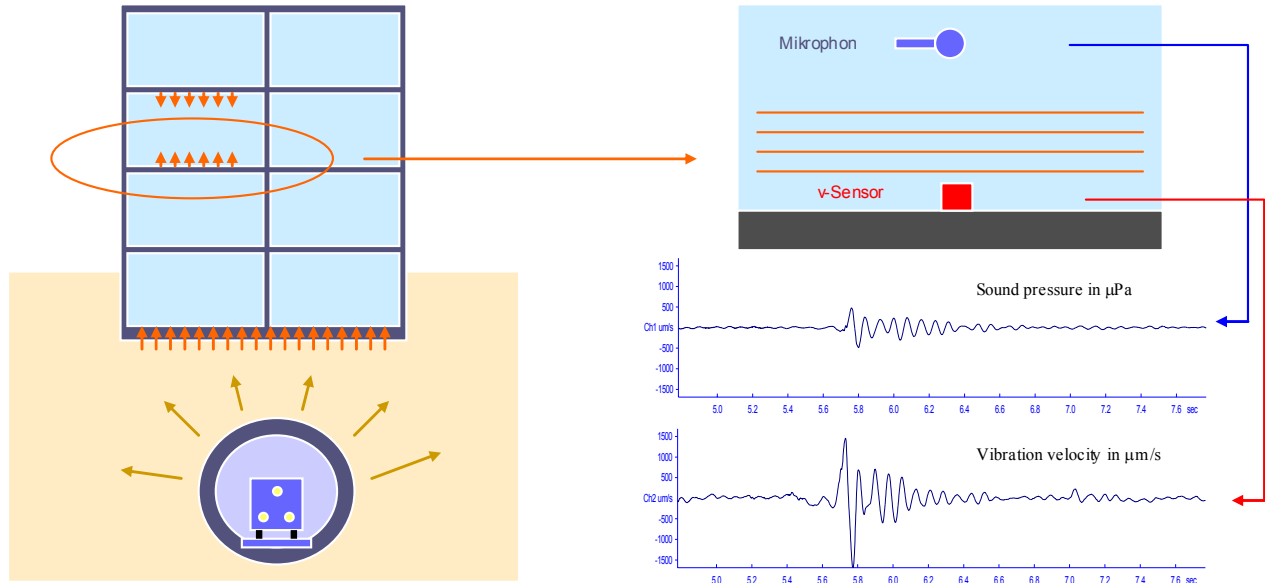


Fig. 2.10a Development of radiated sound in a building above a tunnel

This phenomenon is actually nothing else than the wave propagation in a rod that gets an impulse as shown in Fig. 2.11. The impulse of duration t_n with a pressure of σ_x creates a compressed zone of length x_n that travels at the speed c along the rod. The displacement u is given by Eq. (2.6) and this can be converted into Eq. (2.7) and Eq. (2.8).

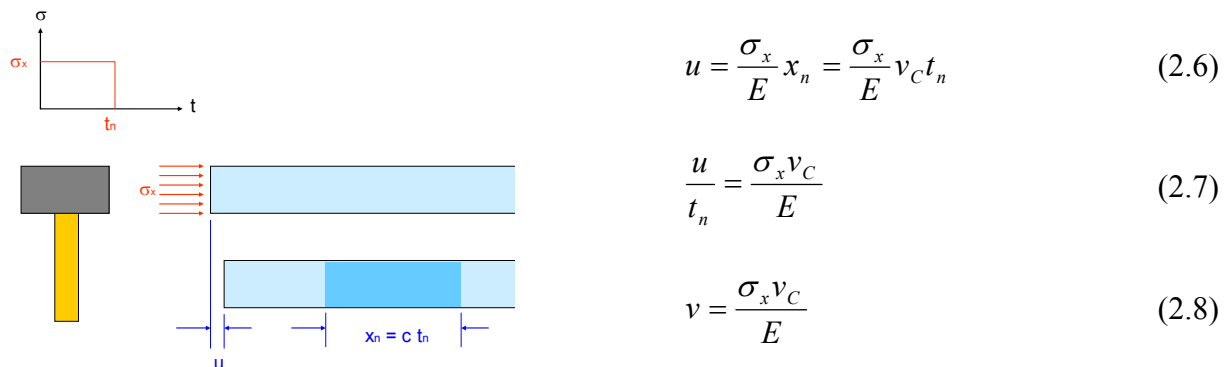


Fig. 2.11 Compression wave in a rod

Using the relationship $E = \rho \cdot v_c^2$ in Eq. (2.8) we obtain the relation between the applied pressure (which is also the pressure in the compressed zone and the particle velocity):

$$\sigma_x = v \cdot \rho \cdot v_c \quad (2.9)$$

With $\rho = 1.2 \text{ kg/m}^3$ and $v_c = 333 \text{ m/s}$ we obtain:

$$p \equiv \sigma_x = 0.4 \cdot v \cdot Pa \cdot \frac{1}{mm/s} \quad (2.10)$$

This means that – based on the theory of the longitudinal wave in a rod - the pressure in the air above the floor can be calculated from the vibration velocity of the floor. Furthermore for normal conditions ($\rho = 1.2 \text{ kg/m}^3$ and $v_c = 333 \text{ m/s}$) the pressure (given in Pa) is 40 % of the floor vibration velocity (given in mm/s). As can be seen in Fig. 2.10, where the velocity and pressure time history caused by a jump on the floor is shown, this correlation can easily be tested.

Many recommendations for the calculation of radiated noise suggest to take the floor vibration value in dBA and to add 6 to 10 dB. This in fact corresponds to the correlation

$$p = 0.8 \cdot \dots \cdot 1.2 \cdot v \cdot Pa \cdot \frac{1}{mm/s} \quad (2.11)$$

i.e. the effective radiated sound level is 2 to 3 times higher than in the theoretical case given in Eq. (2.10). This might well be explained by the fact that not only the floor but also the ceiling and the walls contribute to the radiated noise.

3 Modelling of Railway Vibrations

3.1 VIBRA-1: A frequency-independent Model

VIBRA-1 is an easy to use computer program for the analysis of train induced vibrations and ground borne noise. Using readily available data on train traffic, track type, track subsoil and on the structure of the adjacent buildings the ground borne vibrations and noise are computed. The analysis is based on a semi-empirical model that combines the theory of wave propagation with data from a large number of measurements of ground borne vibration and noise.

3.1.1 Vibrations

Fig. 3.1 shows the basic principles of train induced vibrations: Vibrations created by passing trains are propagated through the soil to the building foundation. From there they travel through the walls up to the floors and ceilings in the entire building. On its path from the track to the building foundation the vibrations are attenuated in the soil due to geometric and material damping. While passing from the soil to the building foundation the vibrations are considerably attenuated due to the so called “coupling effect”. The vibrations reach the higher storeys through the walls without too much modification. In the floors the vibrations are generally amplified by a large degree due to resonance phenomena.

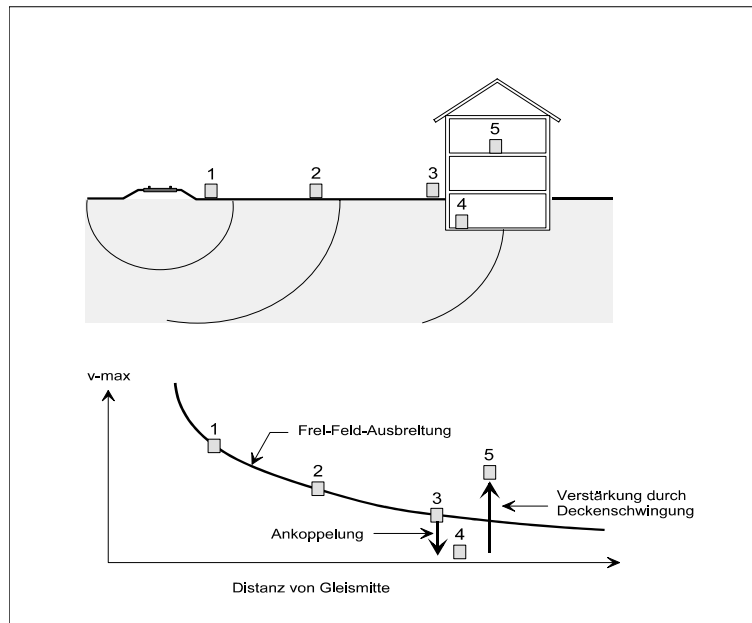


Fig. 3.1 Schematic diagram of vibration propagation

The propagation behaviour shown in Fig. 3.1 can be expressed by the following equation:

$$v = v_0 \cdot F_r \cdot F_z \left(\frac{G}{G_0} \right)^h F_s \left(\frac{r_0}{r} \right)^m F_a \cdot F_e \quad (3.1)$$

v = RMS-value (over train passage time) of the vibration velocity in the middle of the floor.

v_0 = Reference value of vibration. This value corresponds to the characteristic vibration in a distance r_0 (= reference distance) from the track centre line.

- F_r = Factor for reference value v_0 . Basically VIBRA-1 uses the RMS-value of vibration velocity as the characteristic vibration value. However the value for v_0 can also be defined in terms of KB or as another characteristic value derived from vibration velocity. With F_r these different definitions for the reference value are taken into account.
- F_z = Weighting factor for train type. For instance freight trains would typically get a higher weighting factor than passenger trains.
- G = Average velocity of a train type.
- G_0 = Reference train velocity
- h = Exponent for scaling train velocity
- F_s = Track factor for switches and other track irregularities
- r_0 = Reference distance
- r = Distance between building and track centre line
- m = Exponent for geometric and material damping in the soil
- F_a = Coupling factor between building and soil
- F_e = Factor for vibration amplification in floor

3.1.2 Ground borne noise

Ground borne noise is created by the vibration of floor and ceiling and to a lesser extent by the vibration of walls. The correlation between vibration amplitude in a room and ground borne noise (or more accurately: secondary radiated sound) is quite complex. For an infinite rigid plate the conditions are still comparatively straightforward. The correlation between radiated sound and vibration of the rigid plate has been discussed in 2.5. Using Decibel expressions it can be expressed in the following form:

$$L_p = L_v + 20 \cdot \log \sigma \quad (3.2)$$

- with:
- L_p = RMS-value of radiated sound in dB ($p_{\text{ref}} = 2 \cdot 10^{-5}$ Pa)
 - L_v = RMS-value of vibration of rigid plate in dB ($v_{\text{ref}} = 5 \cdot 10^{-5}$ mm/s)
 - σ = Radiation efficiency

This means that – with the reference values specified above – the sound pressure in dB is numerically equal to the vibration of the rigid plate plus a term representing the radiation efficiency of the plate. For an infinite rigid plate $\sigma = 1$ and thus the last term in Eq. (3.2) is zero. In a room this term however is not zero. Due to the reflexions and due to the vibrations of the ceiling and the walls this term can reach values as high as 20 dB.

In VIBRA-1 the calculation of ground borne noise is carried out with the following equation:

$$L_{Aeq} = L_v - A + \sigma \quad (3.3)$$

- with
- L_{Aeq} = A-weighted equivalent sound pressure level of a passing train in the middle of the room
 - L_v = RMS value of v over the train passage time for the frequency range between 50 and 125 Hz, converted into dB (with $v_{\text{ref}} = 5 \cdot 10^{-5}$ mm/s). v is calculated with Eq. (3.1) but with the parameters for ground borne noise.
 - A = A-weighting: A value of 26 dB is used, which corresponds to the A-weighting of the 63-Hz band.
 - σ = Factor taking in account the transition from vibration to sound (i.e. secondary radiated sound)

3.2 VIBRA-2: A Spectral Model

As opposed to VIBRA-1, which determines the vibrations using a frequency-independent model and where only limited data about subsoil and building structure are used, VIBRA-2 is based on a rather complex model that includes all important aspects of vibration propagation near railway lines.

As starting point a so called “source spectrum” is used. This corresponds to a third octave spectrum of a specified train type running at a specified velocity on a specified track type on a specified subsoil (or tunnel) measured at a specified distance from the track. Starting from this source spectrum the vibration spectrum for different locations is computed – as shown in Fig. 3.2 – by multiplying the source spectrum successively with corresponding transfer spectra.

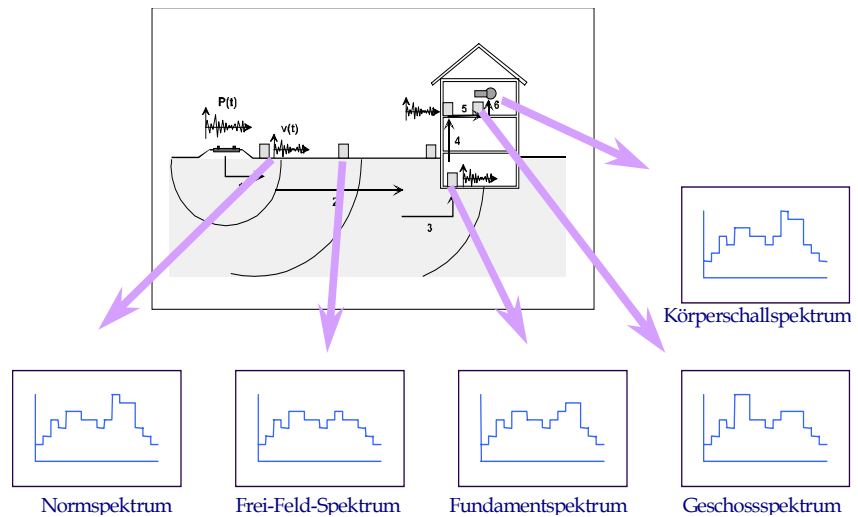


Fig. 3.2 Schematic representation of analysis procedure used in VIBRA-2

From the track spectrum we obtain the free-field spectrum by multiplying the track spectrum with the transfer spectrum for track position (tunnel, embankment, cut) and the transfer spectrum for geometrical and material damping. This is the spectrum we would measure at a measuring point in the free field at the specified distance from the track. If required a transfer spectrum for special free field features (e.g. isolating barriers) could be introduced.

Multiplying the free field spectrum with the transfer spectrum for soil-building coupling will lead to the foundation spectrum. Multiplying the foundation spectrum with the transfer spectrum for the floor yields the floor spectrum. If required a transfer spectrum for special building features (e.g. vibration isolation) can be introduced. From the floor spectrum we calculate the spectrum for ground borne noise. Thus this procedure of successive multiplication of the source spectrum with various transfer spectra yields eventually to the vibration in the room and to the ground borne noise. Fig. 3.3 and 3.4 show the type of result presentation we obtain with VIBRA-2.

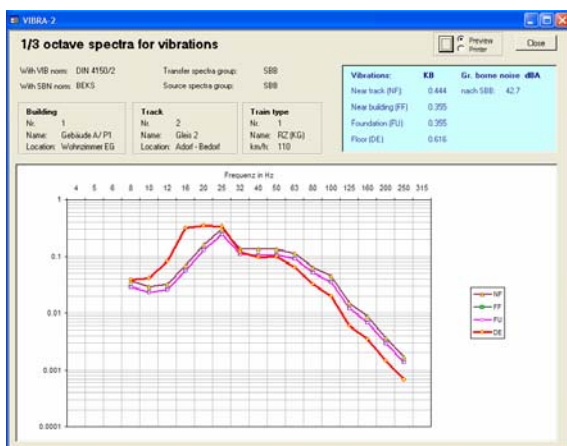


Fig. 3.3 Diagram showing the results for one train type

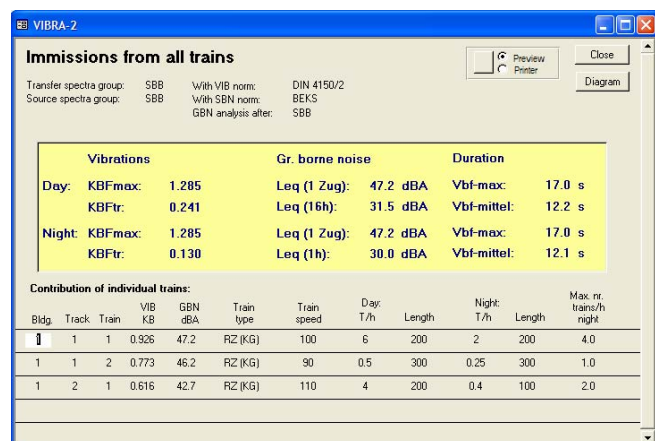


Fig. 3.4 Display of results for all train types

4 VIBRA-1: Comparison with measurements

A measurement project carried out in summer 2005 offered an excellent opportunity to check the results of the VIBRA-1 predictions with actual measurements. The project included the measurement of vibrations and structure borne noise in 10 buildings along a railway line, where recently a third track has been added. Table 4.1 lists the most important features of these 10 buildings and compares the VIBRA-1 predictions with the measured values. With the exception of number 5 and 7 all buildings were typical single family dwellings with two storeys. Number 5 was an old mansion and number 7 an industrial building with a few studios.



Table 4.1 Comparison of VIBRA-1 prediction with measurement

Nr.	Floor type	Distance from track	Switch	VIBRA-1 prediction		Measured values	
				v-rms (mm/s)	L _{Aeq} (dBA)	v-rms (mm/s)	L _{Aeq} (dBA)
0	concrete	18	no	0.100	27.4	0.074	
1	wood	40	yes	0.175	22.8	0.061	21.0
2	wood	12	yes	0.613	35.1	0.064	21.9
3	wood	13	no	0.282	30.8	0.067	15.0
4	concrete	15	yes	0.243	32.8	0.180	27.8
5	wood	5.5	no	0.455	36.0	0.118	19.3
6	concrete	10	yes	0.370	37.0	0.095	21.7
7	concrete	4	no	0.317	39.2	0.030	21.2
8	concrete	55	no	0.031	16.0	0.029	20.7
9	concrete	17	no	0.107	28.0	0.031	

As we can see from Fig. 4.1 the vibration prediction with VIBRA-1 is for all buildings higher than the measured values. This might be a reassuring feature but at the same time we have to ask whether a prediction model should really have a safety margin that is in the average between 2 and 4.

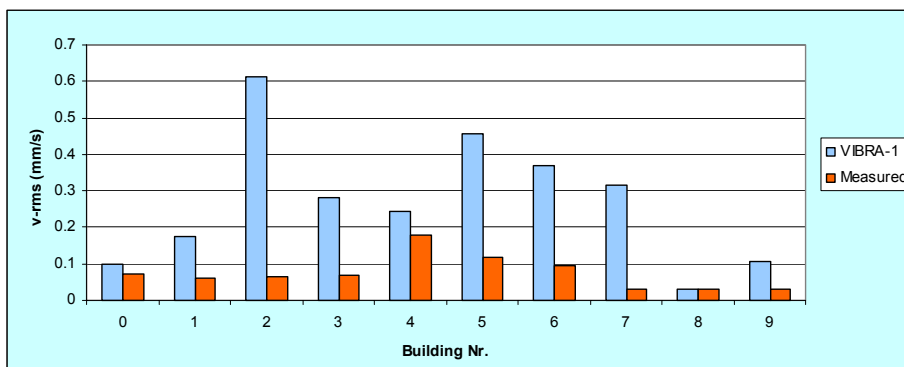


Fig. 4.1 Comparison of VIBRA-1 prediction with measurement for vibration

The same holds for the prediction of structure borne noise as shown in Fig. 4.2. Except for building number 8 the predictions are all on the conservative side, usually 10 to 15 dB higher. However for building number 1 and 4 the safety margin is quite small.

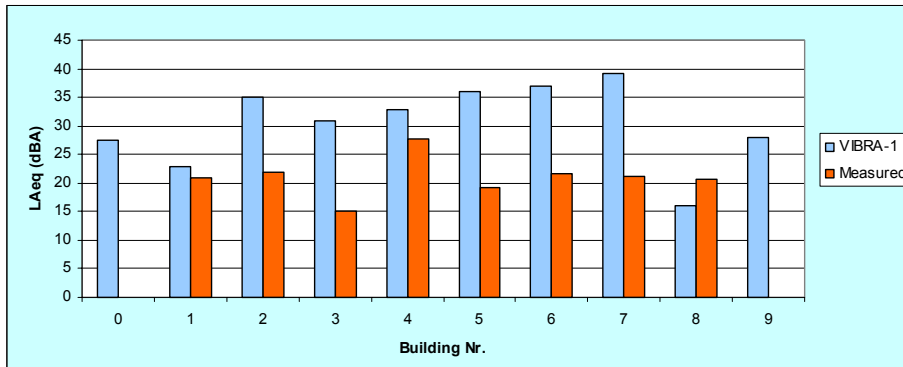


Fig. 4.2 Comparison of VIBRA-1 prediction with measurement for structure borne noise

In order to find the reason for this behaviour of VIBRA-1 the individual steps used in the prediction shall be investigated.

4.1 Prediction of Free-Field vibration

Fig. 4.3 shows the comparison of VIBRA-1 predictions with measurements for free-field vibrations, i.e. for a point in the field having the same distance from the track as the building. With the exception of number 8 the prediction is always higher than the measured value. The largest discrepancy occurs for building 2 and 6. For building 2 the reason for the discrepancy lies in the fact that the faster trains all run on track 2 and 3 and not on track 1 which is closest to building 2. The VIBRA-1 model however assumes that all trains run on track 1. This assumption is obviously too much on the safe side. Most likely this is also the reason why for most buildings the free-field prediction is too high. In the vicinity of building 6 the track runs in a cut which usually reduces the vibrations. This fact is not taken into account in VIBRA-1. The inverse discrepancy for building 8, which is at a distance of 55 m from the track, cannot be explained.

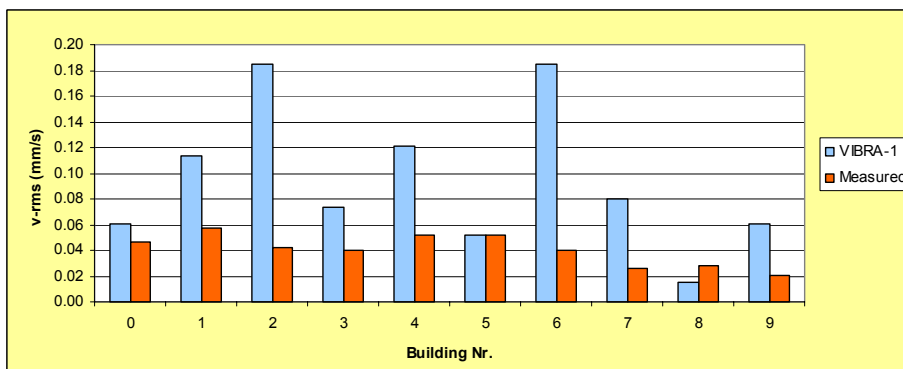


Fig. 4.3 Comparison of VIBRA-1 prediction with measurement for free-field vibration

4.2 Coupling between Soil and Building

VIBRA-1 assumes that “light” building exhibit a coupling factor of 0.5 and “heavy” buildings a factor of 0.33. As we can see from Fig. 4.3 this assumption is quite adequate.

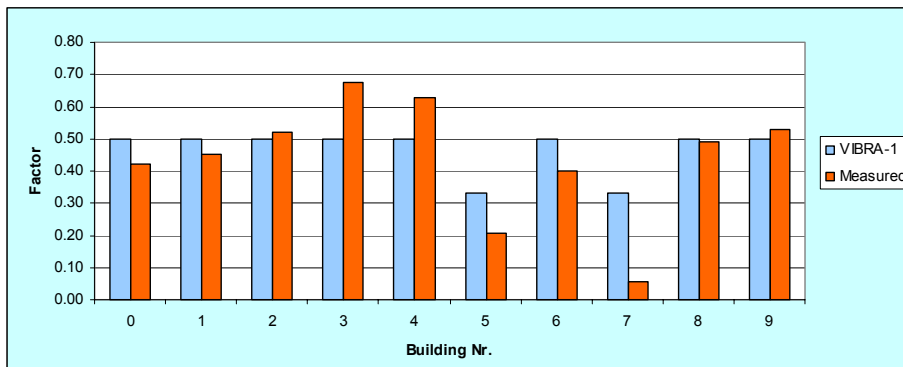


Fig. 4.3 Comparison of VIBRA-1 prediction with measurement for free-field vibration

4.3 Amplification in floors

For the amplification in the floor VIBRA-1 uses a factor of 4 for concrete floors and 8 for wooden floors. A factor of 4 for concrete floors is obviously not on the safe side. The few cases shown in Fig. 4.3 suggest rather a value of 5. The factor for wooden floors might also be set to 5. Recent statistical evaluation of some 50 measurements (each having more than 100 train passages) yields an average value of 4.2 for concrete floors and 3.9 for wooden floors.

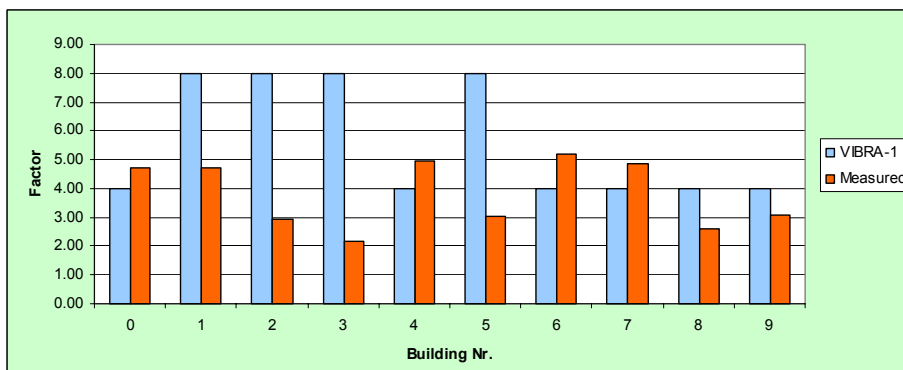


Fig. 4.3 Comparison of VIBRA-1 prediction with measurement for free-field vibration

4.4 Improvements in VIBRA-1

All in all it can be said that the model in VIBRA-1 yields predictions that are – in most cases – on the safe side. The model could be enhanced by introducing multiple tracks, by considering the effect of dams and cuts and by adapting the factors for floor amplification to the latest results from statistical evaluations.

5 VIBRA-2

VIBRA-2 is, as explained in chapter 3.2, a spectral model. Starting with a source spectrum the spectrum for different locations is computed by multiplying the source spectrum successively with appropriate transfer spectra. The model is most useful in situations where part of the spectral data required in the calculation can be obtained by measurements. A typical application for instance is the vibration prediction for a new building. The vibrations at the future site can be measured and the third octave spectrum of this measurement can be introduced into the VIBRA-2 model as source spectrum. Apart of predicting vibrations and structure borne noise the effect of sub-ballast mats or of a building isolation systems can be tested.

5.1 Source spectra

The source spectrum, i.e. the third octave spectrum of the vibration in the free-field at a reference distance (e.g. 8 m from the track) forms the starting point of the calculation. In many cases this spectrum can be measured directly or adapted from measurements on similar sites. VIBRA-2 contains a collection of source spectra for different train types and train velocities, which have been developed by the SBB (Swiss Rail). Figure 5.1 shows two source spectra obtained from 30 different sites with more than 100 trains per site. No distinction has been made for train type. They represent an average train mix with train velocities between 80 and 100 km/h. As expected the spectrum for 5 to 15 m distance exhibits higher amplitudes at higher frequencies while the spectrum for 15 to 25 m is more broad-banded with lower dominant frequencies.

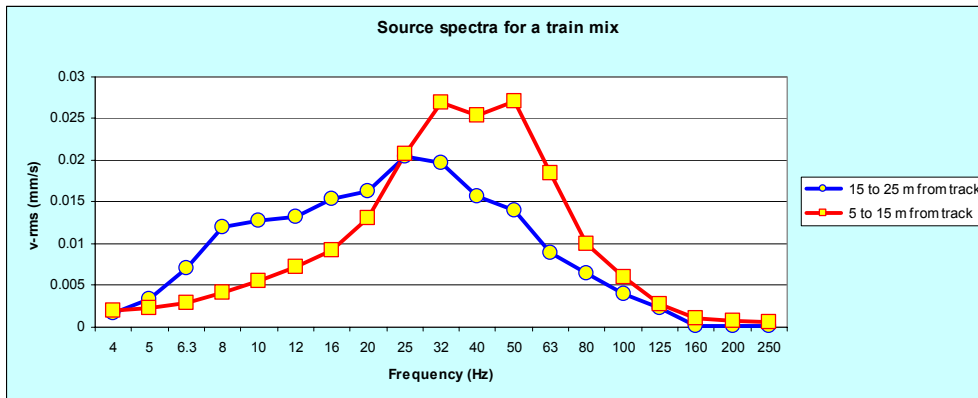


Fig. 5.1 Source spectra for average train mix for two different track distances

5.2 Coupling effect

The transfer spectra in Fig. 5.2 show the coupling effect for “light” and “heavy” buildings. They have been obtained by evaluating 14 sites with 1 – 2 storey (light) buildings and 7 sites with 3 – 6 storey (heavy) buildings. Contrary to our expectation there is no marked difference between the two spectra.

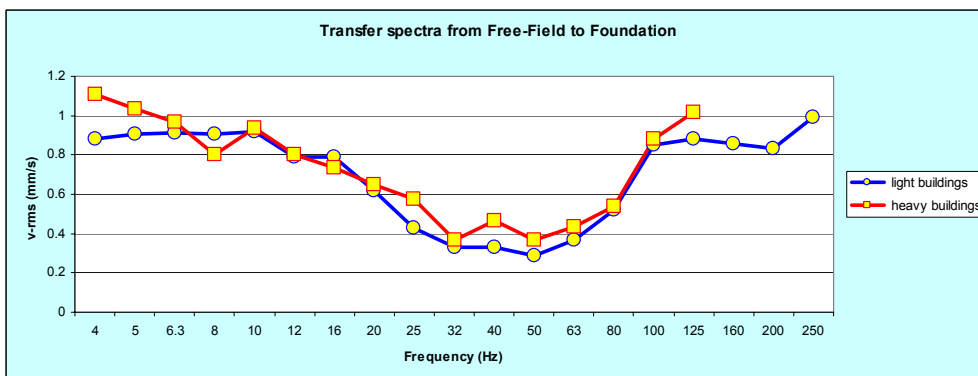


Fig. 5.2 Transfer spectra for the coupling effect for “light” and “heavy” buildings

5.3 Floor transfer spectra

Floor transfer spectra exhibit – as expected – peak amplification at the eigenfrequency of the floor. The spectra shown below have been obtained by evaluating 60 sites with more than 100 trains per site. The peak amplification for timber floor is in the order of 6 to 8 and for concrete floor in the order of 8 to 10. This corresponds quite well with the theoretical model described in chapter 2.4. It should be noted that apart from the amplification in the eigenfrequency there is also amplification in higher frequencies which might be due to higher eigenmodes.

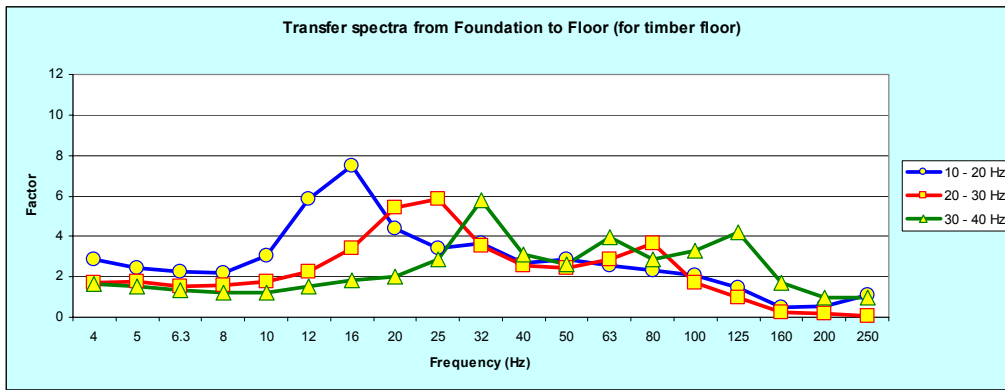


Fig. 5.3 Transfer spectra for timber floors

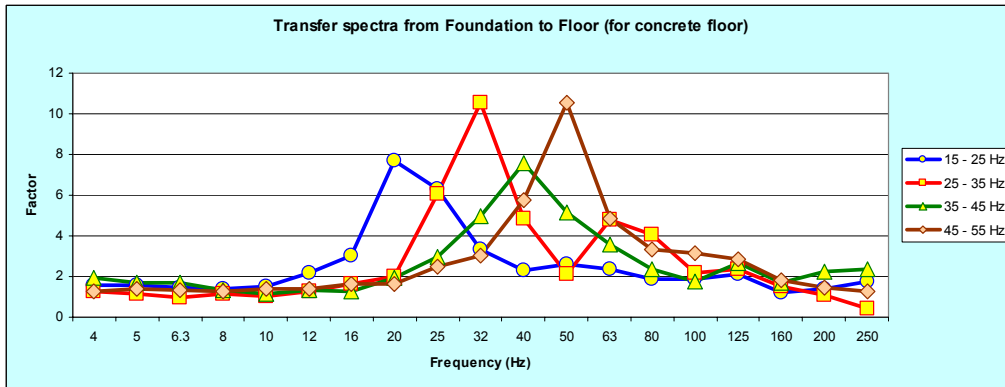


Fig. 5.4 Transfer spectra for concrete floors

5.4 Structure borne noise

The transfer from floor vibration to radiated sound follows – as we have seen in chapter 2.5 – the same law as the compression wave in a rod. For the ideal case, i.e. a rigid plate and no reflexions and absorptions from the ceiling and the walls, we could calculate the radiated sound with:

$$p = 0.4 \cdot v \cdot Pa \cdot \frac{1}{mm / s} \tag{5.1}$$

With the contribution of the ceiling and the walls the radiated sound must certainly be higher. The evaluation of 42 concrete buildings and 14 buildings with timber floors the transfer spectra of Fig. 5.5 have been obtained. The radiation efficiency obviously increases with increasing frequency, starting with 0.7 Pa / mm * s at 40 to 50 Hz and reaching 1.6 Pa / mm * s at 125 Hz.

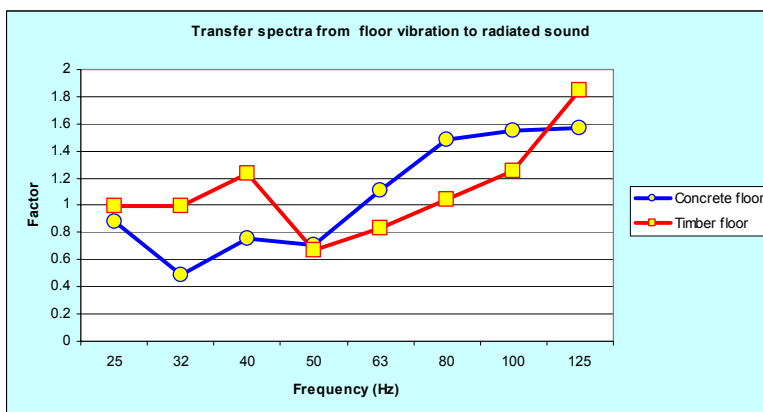


Fig. 5.5 Transfer spectra for radiated sound

6 Conclusions

The comparison of the vibration prediction model used in VIBRA-1 with the measurements in 10 buildings has revealed some interesting results: The model is – as has been experienced by many users of VIBRA-1 – generally on the conservative side. But it is not always on the safe side. In exceptional cases where concrete floors have low damping and an eigenfrequency that coincides with the dominant frequency of the excitation the overall amplification factor can be as high as 7.

To improve the prediction quality with the model in VIBRA-1 the following changes should be introduced:

- The amplification factor for concrete floors should be increased from 4 to 5.
- The amplification factor for timber floors should be reduced from 8 to 5. A distinction between the two floor types seems - on the basis of the available data - not justified.
- For railway lines with multiple tracks the effective track distance should be used. The assumption that all trains run on the closest track leads to over-conservative predictions.
- The influence of the position of the track, i.e. level, dam or cut should be included in the model.

With these improvements VIBRA-1 will hopefully provide vibration predictions that are more reliable and at the same time not too much on the conservative side.